Charge Generation in the Oscillating Background

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Abstract

The preheating after the inflation, which can be interpreted as particle creation in the oscillating inflaton background, provides a state far from thermal equilibrium. We extend the field theoretical treatment of the preheating by Linde et al. to the case of multi-component complex scalars to show that some charges are created in this process, if C and CP are violated. A new possibility of baryogenesis based on this mechanism is also discussed.

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1 Introduction

In the scenario of inflationary cosmology, which can explain the unanswered problems left by the standard big bang scenario, all the particles at present are thought to be created in the reheating process following the exponentially inflating de Sitter phase. That process is viewed as the decay of the inflaton field, which begins to oscillate around the minimum of the inflaton potential. The inflaton itself is treated as a coherently oscillating classical field, and its oscillation is damped by its decay, whose decay products fill the universe to realize the hot universe. A several yeas ago, it was pointed out by Linde et al.[1] that light bosons could be produced more drastically than the perturbative reheating, when some conditions are satisfied on their coupling to the inflaton and the amplitudes and frequency of the inflaton oscillation. This process is called *preheating*, to distinguish from the perturbative reheating.

The preheating is nothing but particle creation in the oscillating background. Although particle creation or annihilation generally occurs in an time-dependent background, the number of created particles grows exponentially when the background is periodic in time. This is a kind of parametric resonance. This process of particle creation is far from equilibrium, and the state after that has large quantum fluctuation composed of low frequency modes, whose relaxation process to the equilibrium is also out of equilibrium. Hence there are chances for the matter-antimatter asymmetry of the universe to be generated in these eras, if C and CP are violated during these processes. In fact, the preheating has been argued in connection to generation of the baryon asymmetry of the universe (BAU)[2], focusing on the fact that the low-frequency quantum fluctuations of the created particles are so large that the sphaleron processes may be enhanced. Besides this possibility, it is natural to think that the baryon number or other charges were generated during the preheating when all the particles in the universe were created. We investigate this possibility to generate a charge asymmetry in the presence of C and CP violation in the oscillating background.

We study the particle creation when C and CP is violated in the interaction of quantum scalar fields with the background. We extend the analysis by Linde et al.[3] to a system of multicomponent complex scalar fields. The interaction with the oscillating background is approximated by the successive scattering with some potential, while the effects of CP violation are evaluated perturbatively. In [3], the number density of the created particles during the preheating was expressed in terms of the wave function in the inflaton background, and the results were reproduced by the picture of successive scattering approximation. It is, however, difficult to distinguish the positive-frequency wave from the negative-frequency wave in the oscillating background, so that one cannot define

a charge, to which the contributions from the antiparticles have opposite sign relative to the particles, in terms of wave functions. Then the treatment based on the successive Bogoliubov transformations will be useful. In Section 2, we generalize the field theory of a real scalar in the oscillating background studied in [3] to the case of multicomponent complex scalars and formulate the descent equations for the Bogoliubov coefficients. We apply the method to a simple toy model and find some charges to be generated, in Section 3. The final section is devoted to discussions.

2   Field Theoretical Treatment

Consider a system of n complex scalar fields interacting with the inflaton through the effective lagrangian

$$\mathcal{L} = \partial_\mu \chi_a^* \partial^\mu \chi_a - m_a^2(t)\chi_a^*\chi_a - \chi_a^* V_{ab}(t)\chi_b - \frac{1}{2}\left(\chi_a W_{ab}(t)\chi_b + \text{c.c.}\right), \qquad (2.1)$$

where $m_a^2(t) = g_a^2\phi^2(t)$ and $a, b = 1, 2, \cdots, n$. The coefficients $V_{ab}(t)$ and $W_{ab}(t)$ represent the collective effects of the oscillating background and interactions with other fields. We treat their effects as perturbation. For simplicity, we consider the case of the undamped background oscillation, with the parameters in the broad resonance regime. That is, we assume that the inflaton field is given by

$$\phi(t) = \Phi \sin mt, \qquad (2.2)$$

where m is the inflaton mass and Φ is the constant amplitude, and neglect the redshift of frequency of each mode. In this case, each mode of the unperturbed wave functions satisfies the mode equation

$$\left(\partial_t^2 + k^2 + g_a^2\Phi^2 \sin^2 mt\right)\chi_{ak}(t) = 0. \qquad (2.3)$$

It is known that wave functions in any periodic potential belong to either stability or instability bands[4]. The solutions to (2.3) are given by the Mathieu functions, which have this property. Among the solutions, those in the instability bands increase their particle numbers exponentially. In particular, for $q_a \equiv \frac{g_a^2\Phi^2}{4m^2} \gg 1$, some modes are in the broad instability bands so that they continue to grow for a long time, even if we take the effects of the expanding universe into account. Further, the frequencies of the modes in the broad resonance band are much larger than that of the inflaton oscillation. Then, the particle number changes only when the inflaton field vanishes, while the χ-fields evolve adiabatically for time intervals with $|\phi(t)| \neq 0$. This observation motivates the study of preheating by approximating the oscillating background as a series of successive

scattering by some simple potential. In [3], the inflaton field near its zero was replaced by the quadratic potential, and the descent equation for the particle number density in some mode was derived. The particle number density is given in terms of the wave function, which is a solution to (2.3) with *any* boundary condition. As for a charge density, we must distinguish the particle from the antiparticle. This amounts to impose the boundary condition defining either positive- or negative-frequency mode. Here we express the densities of particle number and the charges in terms of the Bogoliubov coefficients which relates the operators before the inflaton begins to oscillate and those in the adiabatic time intervals.

2.1 Descent equations for the Bogoliubov coefficients

We assume that the particles are generated only in the short time interval near $t_j = \pi j/m_a$, where j is a non-negative integer, and that the violation of the charge conservation and CP symmetry is effective only within the short interval when $m_a(t) \simeq 0$. Hence, for t with $t_{j-1} \ll t \ll t_j$, $\chi_{ak}(t)$ satisfies the unperturbed equation (2.3). Let $f_{ak}^j(t)$ be the properly normalized positive-frequency solution to (2.3) in the adiabatic interval between t_{j-1} and t_j. Then the χ-field operator is expanded as

$$\chi_a(x) = \int d^3\boldsymbol{k} \left(a_{a\boldsymbol{k}}^j f_{ak}^j(t) e^{i\boldsymbol{k}\boldsymbol{x}} + b_{a\boldsymbol{k}}^{j\dagger} f_{ak}^{j*}(t) e^{-i\boldsymbol{k}\boldsymbol{x}} \right). \tag{2.4}$$

Without the perturbation, upon scattered by the potential $g_a^2 \phi^2(t)$ at t_j, the positive-frequency wave $f_{ak}^j(t)$ is transmitted to a linear combination of $f_{ak}^{j+1}(t)$ and $f_{ak}^{j+1*}(t)$, whose coefficients are determined by solving the scattering problem by the potential at t_j. In the presence of the perturbation V_{ab} and W_{ab}, the scattered wave is composed of various components of $f_{ak}^{j+1}(t)$ and $f_{ak}^{j+1*}(t)$. Suppose that the positive-frequency wave $f_{ak}^0(t)$ at $t < t_0$ transfers to a set of the positive- and negative-frequency waves in the j-th interval;

$$\begin{pmatrix} 0 \\ \vdots \\ f_{ak}^0(t) \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} \alpha_{a1}^j f_{1k}^j(t) + \beta_{a1}^j f_{1k}^{j*}(t) \\ \vdots \\ \alpha_{ab}^j f_{bk}^j(t) + \beta_{ab}^j f_{bk}^{j*}(t) \\ \vdots \\ \alpha_{an}^j f_{nk}^j(t) + \beta_{an}^j f_{nk}^{j*}(t) \end{pmatrix}, \tag{2.5}$$

while the negative-frequency wave $f_{ak}^{0*}(t)$ transfers as

$$\begin{pmatrix} 0 \\ \vdots \\ f_{ak}^{0*}(t) \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} \tilde{\beta}_{a1}^j f_{1k}^j(t) + \tilde{\alpha}_{a1}^j f_{1k}^{j*}(t) \\ \vdots \\ \tilde{\beta}_{ab}^j f_{bk}^j(t) + \tilde{\alpha}_{ab}^j f_{bk}^{j*}(t) \\ \vdots \\ \tilde{\beta}_{an}^j f_{nk}^j(t) + \tilde{\alpha}_{an}^j f_{nk}^{j*}(t) \end{pmatrix}, \tag{2.6}$$

4

where $\alpha_{ab}^j \neq \tilde{\alpha}_{ab}^j$ and $\beta_{ab}^j \neq \tilde{\beta}_{ab}^j$ because of the CP violation. Then the Bogoliubov transformation between the creation and annihilation operators at $t < t_0$ and those at $t_{j-1} < t < t_j$ is

$$
\begin{aligned}
a_{a\boldsymbol{k}}^j &= a_{b\boldsymbol{k}}^0 \alpha_{ba}^j + b_{b\boldsymbol{k}}^{0\dagger} \tilde{\beta}_{ba}^j, \\
b_{a\boldsymbol{k}}^{j\dagger} &= a_{b\boldsymbol{k}}^0 \beta_{ba}^j + b_{b\boldsymbol{k}}^{0\dagger} \tilde{\alpha}_{ba}^j.
\end{aligned}
\tag{2.7}
$$

Requiring

$$
\left[a_{a\boldsymbol{k}}^j, a_{b\boldsymbol{k'}}^{j\dagger}\right] = \left[b_{a\boldsymbol{k}}^j, b_{b\boldsymbol{k'}}^{j\dagger}\right] = \delta_{ab}\delta^3(\boldsymbol{k} - \boldsymbol{k'}), \qquad \left[a_{a\boldsymbol{k}}^j, b_{b\boldsymbol{k'}}^j\right] = 0,
\tag{2.8}
$$

we find the conditions on the Bogoliubov coefficients:

$$
\begin{aligned}
\alpha_{ca}^j \alpha_{cb}^{j*} - \tilde{\beta}_{ca}^j \tilde{\beta}_{cb}^{j*} &= \delta_{ab}, \\
\tilde{\alpha}_{ca}^{j*} \tilde{\alpha}_{cb}^j - \beta_{ca}^{j*} \beta_{cb}^j &= \delta_{ab}, \\
\alpha_{ca}^j \beta_{cb}^{j*} - \tilde{\beta}_{ca}^j \tilde{\alpha}_{cb}^{j*} &= 0.
\end{aligned}
\tag{2.9}
$$

In terms of the n-by-n matrix notation, $\alpha^j = \left(\alpha_{ab}^j\right)$, $\beta^j = \left(\beta_{ab}^j\right)$, etc., these conditions are expressed as

$$
\alpha^{j\dagger}\alpha^j - \tilde{\beta}^{j\dagger}\tilde{\beta}^j = \tilde{\alpha}^{j\dagger}\tilde{\alpha}^j - \beta^{j\dagger}\beta^j = 1, \qquad \beta^{j\dagger}\alpha^j - \tilde{\alpha}^{j\dagger}\tilde{\beta}_j = 0.
\tag{2.10}
$$

We can also show that

$$
\alpha^j\alpha^{j\dagger} - \beta^j\beta^{j\dagger} = \tilde{\alpha}^j\tilde{\alpha}^{j\dagger} - \tilde{\beta}^j\tilde{\beta}^{j\dagger} = 1, \qquad \alpha^j\tilde{\beta}^{j\dagger} - \beta^j\tilde{\alpha}^{j\dagger} = 0.
\tag{2.11}
$$

Now we define the number density of the particles with momentum $\boldsymbol{k}$ in the j-th interval as

$$
n_k^j \equiv \frac{1}{V}\langle 0^0| \sum_{a=1}^n \left(a_{a\boldsymbol{k}}^{j\dagger} a_{a\boldsymbol{k}}^j + b_{a\boldsymbol{k}}^{j\dagger} b_{a\boldsymbol{k}}^j\right) |0^0\rangle,
\tag{2.12}
$$

where $|0^0\rangle$ is defined by $a_{a\boldsymbol{k}}^0|0^0\rangle = b_{a\boldsymbol{k}}^0|0^0\rangle = 0$ and $V = \delta^3(\boldsymbol{k} = \boldsymbol{0})$. By use of (2.7), we have

$$
n_k^j = \sum_{a,b} \left(\tilde{\beta}_{ba}^{j*}\tilde{\beta}_{ba}^j + \beta_{ba}^j\beta_{ba}^{j*}\right) = \mathrm{Tr}\left(\tilde{\beta}^{j\dagger}\tilde{\beta}^j + \beta^{j\dagger}\beta^j\right).
\tag{2.13}
$$

If each component of $\phi_a(x)$ has the $U(1)$ charge Q_a, the density of the generated charge in mode $\boldsymbol{k}$ in the j-th interval is given by

$$
j_k^j \equiv \frac{1}{V}\langle 0^0| \sum_{a=1}^n Q_a \left(a_{a\boldsymbol{k}}^{j\dagger} a_{a\boldsymbol{k}}^j - b_{a\boldsymbol{k}}^{j\dagger} b_{a\boldsymbol{k}}^j\right) |0^0\rangle,
\tag{2.14}
$$

which is written in terms of the Bogoliubov coefficients as

$$
j_k^j = \sum_{a,b} Q_a \left(\tilde{\beta}_{ba}^{j*}\tilde{\beta}_{ba}^j - \beta_{ba}^j\beta_{ba}^{j*}\right) = \mathrm{Tr}\left[Q\left(\tilde{\beta}^{j\dagger}\tilde{\beta}^j - \beta^{j\dagger}\beta^j\right)\right],
\tag{2.15}
$$

where Q in the last line is n-by-n matrix, $Q = \mathrm{diag}(Q_1, Q_2, \cdots, Q_n)$.

In the case of $n = 1$, it is easy to find $|\beta^j|^2 = \left|\tilde{\beta}^j\right|^2$ by use of (2.10), so that $j_k^j = 0$. Hence, we need at least two complex scalars interacting with the oscillating background in order to generate charge asymmetry. This fact can be understood as follows: If we regard the particles and the charges as the decay products of the oscillating field, at least two decay channels are needed to produce nonzero charge since the effect of CP violation appears as the interference of the two channels. This is the same situation as the GUT-baryogenesis by the heavy boson decay.

The Bogoliubov coefficients in the $(j + 1)$-th interval are related to those in the j-th interval by solving the scattering problem by the potential at t_j, just as done in [3]. Within each adiabatic interval, the positive frequency wave function $f_{ak}^j(t)$ is well approximated by

$$h_{ak}(t) = \frac{1}{\sqrt{2\omega_a(t)}} e^{-i \int_0^t dt' \, \omega_a(t')}, \tag{2.16}$$

where

$$\omega_a(t) = \sqrt{k^2 + m_a^2(t)} = \sqrt{k^2 + g_a^2 \Phi^2 \sin^2 mt}, \tag{2.17}$$

up to some constant phase. On the other hand, for t near t_j, for which the adiabatic approximation is no longer valid, we solve the scattering problem employing some approximate form of the potential at t_j. We adopt another type of potential which allows to define an asymptotic state, that is, $\omega_a(t) \simeq \omega_a^0 = \mathrm{const.}$ for $|t - t_j| \gg 1/m$, in contrast to the quadratic potential $m_a^2(t) \simeq m^2(t - t_j)^2$ used in [3]. This choice of the potential is more appropriate for the perturbative calculation of the CP-violating scattering presented in the next subsection. The general formulas presented in this subsection holds independent of the detailed form of the potential. If we use this potential, the adiabatic wave (2.16) behaves, far from the potential, as

$$h_{ak}(t) \; \rightarrow \; \frac{1}{\sqrt{2\omega_a^0}} e^{-i\omega_a^0(t - t_j) - i\theta_{ak}^j - i\vartheta_{ak}^\pm}, \qquad (\text{as } t - t_j \rightarrow \pm\infty) \tag{2.18}$$

where $\theta_{ak}^j = \int_0^{t_j} dt' \, \omega_a(t')$ is the phase accumulated by the moment t_j and ϑ_{ak}^+ (ϑ_{ak}^-) is the phase difference between $\int_t^{t_j} dt' \, \omega_a(t')$ and $\omega_a^0(t - t_j)$ for $t \gg t_j$ ($t \ll t_j$). Then the positive-frequency wave at $t < t_0$, $f_{ak}^0(t)$, can be approximated by $h_{ak}(t)$, while $f_{ak}^j(t)$ in the j-th adiabatic interval will be expressed as a linear combination of $h_{bk}(t)$ and $h_{bk}^*(t)$.

Suppose that we find the solutions to the wave equations (usually, by perturbative method) satisfying the boundary condition that only the positive-frequency wave in the

6

a-th component exists at $t \ll t_j$;

$$
\begin{pmatrix} 0 \\ \vdots \\ \frac{1}{\sqrt{2\omega_a^0}} e^{-i\omega_a^0(t-t_j)} \\ \vdots \\ 0 \end{pmatrix} \overset{t \ll t_j}{\longleftarrow} \begin{pmatrix} \chi_{1k}(t) \\ \vdots \\ \chi_{ak}(t) \\ \vdots \\ \chi_{nk}(t) \end{pmatrix} \overset{t \gg t_j}{\longrightarrow} \begin{pmatrix} \mathcal{A}_{a1} \frac{1}{\sqrt{2\omega_1^0}} e^{-i\omega_1^0(t-t_j)} + \mathcal{B}_{a1} \frac{1}{\sqrt{2\omega_1^0}} e^{i\omega_1^0(t-t_j)} \\ \vdots \\ \mathcal{A}_{ab} \frac{1}{\sqrt{2\omega_b^0}} e^{-i\omega_b^0(t-t_j)} + \mathcal{B}_{ab} \frac{1}{\sqrt{2\omega_b^0}} e^{i\omega_b^0(t-t_j)} \\ \vdots \\ \mathcal{A}_{an} \frac{1}{\sqrt{2\omega_n^0}} e^{-i\omega_n^0(t-t_j)} + \mathcal{B}_{an} \frac{1}{\sqrt{2\omega_n^0}} e^{i\omega_n^0(t-t_j)} \end{pmatrix},
$$
$$(2.19)$$

and similarly the solution satisfying the boundary condition that only the negative-frequency wave exists at $t \ll t_j$;

$$
\begin{pmatrix} 0 \\ \vdots \\ \frac{1}{\sqrt{2\omega_a^0}} e^{i\omega_a^0(t-t_j)} \\ \vdots \\ 0 \end{pmatrix} \overset{t \ll t_j}{\longleftarrow} \begin{pmatrix} \chi_{1k}(t) \\ \vdots \\ \chi_{ak}(t) \\ \vdots \\ \chi_{nk}(t) \end{pmatrix} \overset{t \gg t_j}{\longrightarrow} \begin{pmatrix} \tilde{\mathcal{B}}_{a1} \frac{1}{\sqrt{2\omega_1^0}} e^{-i\omega_1^0(t-t_j)} + \tilde{\mathcal{A}}_{a1} \frac{1}{\sqrt{2\omega_1^0}} e^{i\omega_1^0(t-t_j)} \\ \vdots \\ \tilde{\mathcal{B}}_{ab} \frac{1}{\sqrt{2\omega_b^0}} e^{-i\omega_b^0(t-t_j)} + \tilde{\mathcal{A}}_{ab} \frac{1}{\sqrt{2\omega_b^0}} e^{i\omega_b^0(t-t_j)} \\ \vdots \\ \tilde{\mathcal{B}}_{an} \frac{1}{\sqrt{2\omega_n^0}} e^{-i\omega_n^0(t-t_j)} + \tilde{\mathcal{A}}_{an} \frac{1}{\sqrt{2\omega_n^0}} e^{i\omega_n^0(t-t_j)} \end{pmatrix}.
$$
$$(2.20)$$

Here the coefficients $\mathcal{A}$, $\mathcal{B}$, $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$, which are also kinds of Bogoliubov coefficients, satisfy the same relations as α, β, $\tilde{\alpha}$ and $\tilde{\beta}$:

$$
\begin{aligned}
\mathcal{A}^\dagger \mathcal{A} - \tilde{\mathcal{B}}^\dagger \tilde{\mathcal{B}} = \tilde{\mathcal{A}}^\dagger \tilde{\mathcal{A}} - \mathcal{B}^\dagger \mathcal{B} = 1, && \mathcal{B}^\dagger \mathcal{A} - \tilde{\mathcal{A}}^\dagger \tilde{\mathcal{B}} = 0, \\
\mathcal{A}\mathcal{A}^\dagger - \mathcal{B}\mathcal{B}^\dagger = \tilde{\mathcal{A}}\tilde{\mathcal{A}}^\dagger - \tilde{\mathcal{B}}\tilde{\mathcal{B}}^\dagger = 1, && \mathcal{A}\tilde{\mathcal{B}}^\dagger - \mathcal{B}\tilde{\mathcal{A}}^\dagger = 0.
\end{aligned}
$$
$$(2.21)$$

Inverting (2.5) and (2.6) and replacing $f_{ak}^0(t)$ with $h_{ak}(t)$, we have

$$
\begin{pmatrix} 0 \\ \vdots \\ f_{ak}^j(t) \\ \vdots \\ 0 \end{pmatrix} \sim \begin{pmatrix} \left(\alpha^{j\dagger}\right)_{a1} h_{1k}(t) - \left(\tilde{\beta}^{j\dagger}\right)_{a1} h_{1k}^*(t) \\ \vdots \\ \left(\alpha^{j\dagger}\right)_{ab} h_{bk}(t) - \left(\tilde{\beta}^{j\dagger}\right)_{ab} h_{bk}^*(t) \\ \vdots \\ \left(\alpha^{j\dagger}\right)_{an} h_{nk}(t) - \left(\tilde{\beta}^{j\dagger}\right)_{an} h_{nk}^*(t) \end{pmatrix},
$$
$$(2.22)$$

and

$$
\begin{pmatrix} 0 \\ \vdots \\ g_{ak}^{j*}(t) \\ \vdots \\ 0 \end{pmatrix} \sim \begin{pmatrix} \left(\tilde{\alpha}^{j\dagger}\right)_{a1} h_{1k}^*(t) - \left(\beta^{j\dagger}\right)_{a1} h_{1k}(t) \\ \vdots \\ \left(\tilde{\alpha}^{j\dagger}\right)_{ab} h_{bk}^*(t) - \left(\beta^{j\dagger}\right)_{ab} h_{bk}(t) \\ \vdots \\ \left(\tilde{\alpha}^{j\dagger}\right)_{an} h_{nk}^*(t) - \left(\beta^{j\dagger}\right)_{an} h_{nk}(t) \end{pmatrix}.
$$
$$(2.23)$$

Since how $h_{ak}(t)$ and $h_{ak}^*(t)$ appearing in each element in the right-hand sides of (2.22) and (2.23) are scattered by the potential at t_j are determined by (2.19) and (2.20), respectively,

the b-th component of the right-hand side of (2.22) becomes, after the scattering $(t \gg t_j)$,

$$
\left[\left(\alpha^{j\dagger}\right)_{ac} e^{-i(\theta^j_{ck}+\vartheta^-_{ck})}\mathcal{A}_{cb} - \left(\tilde{\beta}^{j\dagger}\right)_{ac} e^{i(\theta^j_{ck}+\vartheta^-_{ck})}\tilde{\mathcal{B}}_{cb}\right] e^{i(\theta^j_{bk}+\vartheta^+_{bk})} h_{bk}(t)
$$
$$
- \left[\left(\tilde{\beta}^{j\dagger}\right)_{ac} e^{i(\theta^j_{ck}+\vartheta^-_{ck})}\tilde{\mathcal{A}}_{cb} - \left(\alpha^{j\dagger}\right)_{ac} e^{-i(\theta^j_{ck}+\vartheta^-_{ck})}\mathcal{B}_{cb}\right] e^{-i(\theta^j_{bk}+\vartheta^+_{bk})} h^*_{bk}(t) \quad (2.24)
$$

which should be identified with $\left(\alpha^{j+1\dagger}\right)_{ab} h_{bk}(t) - \left(\tilde{\beta}^{j+1\dagger}\right)_{ab} h^*_{bk}(t)$, and the b-th component of the right-hand-side of (2.23) becomes

$$
\left[\left(\tilde{\alpha}^{j\dagger}\right)_{ac} e^{i(\theta^j_{ck}+\vartheta^-_{ck})}\tilde{\mathcal{B}}_{cb} - \left(\beta^{j\dagger}\right)_{ac} e^{-i(\theta^j_{ck}+\vartheta^-_{ck})}\mathcal{A}_{cb}\right] e^{i(\theta^j_{bk}+\vartheta^+_{bk})} h_{bk}(t)
$$
$$
- \left[\left(\beta^{j\dagger}\right)_{ac} e^{-i(\theta^j_{ck}+\vartheta^-_{ck})}\mathcal{B}_{cb} - \left(\tilde{\alpha}^{j\dagger}\right)_{ac} e^{i(\theta^j_{ck}+\vartheta^-_{ck})}\tilde{\mathcal{A}}_{cb}\right] e^{-i(\theta^j_{bk}+\vartheta^+_{bk})} h^*_{bk}(t). \quad (2.25)
$$

which should be identified with $\left(\tilde{\alpha}^{j+1\dagger}\right)_{ab} h^*_{bk}(t) - \left(\beta^{j+1\dagger}\right)_{ab} h_{bk}(t)$. If we denote the n-by-n matrix of the phase factors as

$$
U_\pm = \mathrm{diag}\left(e^{-i(\theta^j_{1k}+\vartheta^\pm_{1k})}, \cdots, e^{-i(\theta^j_{nk}+\vartheta^\pm_{nk})}\right), \quad (2.26)
$$

we obtain the descent equations for the Bogoliubov coefficients

$$
\begin{aligned}
\alpha^{j+1} &= U_+\left(\mathcal{A}^\dagger U^\dagger_- \alpha^j - \tilde{\mathcal{B}}^\dagger U_- \tilde{\beta}^j\right) \\
\beta^{j+1} &= U_+\left(\mathcal{A}^\dagger U^\dagger_- \beta^j - \tilde{\mathcal{B}}^\dagger U_- \tilde{\alpha}^j\right) \\
\tilde{\alpha}^{j+1} &= U^\dagger_+\left(\tilde{\mathcal{A}}^\dagger U_- \tilde{\alpha}^j - \mathcal{B}^\dagger U^\dagger_- \beta^j\right) \\
\tilde{\beta}^{j+1} &= U^\dagger_+\left(\tilde{\mathcal{A}}^\dagger U_- \tilde{\beta}^j - \mathcal{B}^\dagger U^\dagger_- \alpha^j\right).
\end{aligned} \quad (2.27)
$$

As a check, one can show that these $(j+1)$-th Bogoliubov coefficients satisfies the same relations as (2.10) and (2.11).

2.2 Perturbative calculation of the scattering matrices

Now we calculate the scattering matrices $\mathcal{A}$, $\mathcal{B}$, $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ by the perturbative method. The wave equations to be solved are written as

$$
\left[\left(\partial^2_t + k^2 + m^2_a(t)\right)\mathbf{1} + \begin{pmatrix} V_{ab}(t) & W^*_{ab}(t) \\ W_{ab}(t) & V^*_{ab}(t) \end{pmatrix}\right]\begin{pmatrix} \chi_{bk}(t) \\ \chi^*_{bk}(t) \end{pmatrix} = 0, \quad (2.28)
$$

For convenience, let us introduce the following symbols:

$$
\begin{aligned}
X_k(t) &= \begin{pmatrix} \chi_{ak}(t) \\ \chi^*_{ak}(t) \end{pmatrix}, \\
\Delta_0 &= \left(\partial^2_t + k^2 + m^2_a(t)\right)\mathbf{1} = \left(\partial^2_t + \omega^2_a(t)\right)\mathbf{1}, \\
\Delta_1 &= \begin{pmatrix} V_{ab}(t) & W^*_{ab}(t) \\ W_{ab}(t) & V^*_{ab}(t) \end{pmatrix}.
\end{aligned} \quad (2.29)
$$

8

Then a perturbative solution to the equation

$$(\Delta_0 + \Delta_1) X_k(t) = 0, \tag{2.30}$$

to the first order with respect to Δ_1 is given by

$$X_k(t) = X_k^{(0)}(t) - \Delta_0^{-1} \Delta_1 X_k^{(0)}(t), \tag{2.31}$$

where $X_k^{(0)}(t)$ is a solution to

$$\Delta_0 X_k^{(0)}(t) = 0. \tag{2.32}$$

In order to evaluate the scattering matrices $\mathcal{A}$ and $\mathcal{B}$, we must prepare $X_k(t)$ such that only $\chi_{ak}(t)$ is the positive-frequency wave and the other components vanish in the remote past. We solve the scattering problem with this initial condition for the approximate potential which admits the asymptotic states. Let $f_{ak}(t)$ be the wave function satisfying

$$\left(\partial_t^2 + \omega_a^2(t)\right) f_{ak}(t) = 0, \quad \text{and} \quad f_{ak}(t) \overset{t \to -\infty}{\longrightarrow} \frac{1}{\sqrt{2\omega_a^0}} e^{-i\omega_a^0 t}. \tag{2.33}$$

Suppose that we solve this unperturbed equation and find the expression as

$$f_{ak}(t) = \alpha_a f_{ak}^{\text{out}}(t) + \beta_a f_{ak}^{\text{out}*}(t), \tag{2.34}$$

where $f_{ak}^{\text{out}}(t)$ satisfies

$$\left(\partial_t^2 + \omega_a^2(t)\right) f_{ak}^{\text{out}}(t) = 0, \quad \text{and} \quad f_{ak}^{\text{out}}(t) \overset{t \to \infty}{\longrightarrow} \frac{1}{\sqrt{2\omega_a^0}} e^{-i\omega_a^0 t}. \tag{2.35}$$

According to the general theory of distorted-wave Born approximation[5], the Green function $G_a(t, t')$ satisfying

$$\left(\partial_t^2 + \omega_a^2(t)\right) G_a(t, t') = \delta(t - t'), \tag{2.36}$$

with the boundary condition such that there is only positive-frequency wave in the remote past is given by

$$G_a(t, t') = \begin{cases} \frac{1}{w} f_{ak}(t) f_{ak}^*(t'), & (t < t') \\ \frac{1}{w} f_{ak}(t') f_{ak}^*(t), & (t' < t) \end{cases} \tag{2.37}$$

where w is the Wronskian defined by

$$w \equiv f_{ak}(t) \partial_t f_{ak}^*(t) - f_{ak}^*(t) \partial_t f_{ak}(t), \tag{2.38}$$

which is time-independent and $w = i$ in our case. Then our Δ_0^{-1} is

$$\Delta_0^{-1} = \text{diag} \left(G_1, \cdots, G_n, G_1^*, \cdots, G_n^* \right). \tag{2.39}$$

9

As the unperturbed wave $X_k^{(0)}(t)$, we take

$$X_k^{[a](0)}(t) = \begin{pmatrix} cf_k(t) \\ c^* f_k^*(t) \end{pmatrix} = \begin{pmatrix} c_{a1} f_{1k}(t) \\ \vdots \\ c_{a1}^* f_{1k}^*(t) \\ \vdots \end{pmatrix}, \tag{2.40}$$

and chose the set of parameters c_{ab} such that

$$X_k^{[a]}(t) = X_k^{[a](0)}(t) - \Delta_0^{-1} \Delta_1 X_k^{[a](0)}(t) \overset{t \to -\infty}{\longrightarrow} \begin{pmatrix} 0 \\ \vdots \\ f_{ak}(t) \\ \vdots \\ f_{ak}^*(t) \\ \vdots \\ 0 \end{pmatrix}. \tag{2.41}$$

In terms of the n-component wave function, the perturbed wave becomes

$$\chi_k^{[a]}(t) = cf_k(t) - GV(cf_k)(t) - GW^*(c^* f_k^*)(t). \tag{2.42}$$

Requiring the initial condition (2.41), we find that c_{ab} should be taken to satisfy

$$c_{ab} + ic_{ac} \int_{-\infty}^{\infty} dt'\, V_{bc}(t') f_{bk}^*(t') f_{ck}(t') + ic_{ac}^* \int_{-\infty}^{\infty} dt'\, W_{bc}^*(t') f_{bk}^*(t') f_{ck}^*(t') = \delta_{ab}. \tag{2.43}$$

Once c_{ab} is determined, we obtain the scattering matrices $\mathcal{A}$ and $\mathcal{B}$ by extracting the coefficients of the positive- and negative-frequency waves, respectively, from (2.42) in the remote future:

$$\begin{aligned} \mathcal{A}_{ab} &= c_{ab}\alpha_b + i\beta_b^* \int_{-\infty}^{\infty} dt'\, f_{bk}(t') \left[V_{bc}(t') c_{ac} f_{ck}(t') + W_{bc}^*(t') c_{ac}^* f_{ck}^*(t') \right], \\ \mathcal{B}_{ab} &= c_{ab}\beta_b + i\alpha_b^* \int_{-\infty}^{\infty} dt'\, f_{bk}(t') \left[V_{bc}(t') c_{ac} f_{ck}(t') + W_{bc}^*(t') c_{ac}^* f_{ck}^*(t') \right]. \end{aligned} \tag{2.44}$$

Similarly, by preparing the negative-frequency wave in the remote past, we can find the other set of the scattering matrices $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$. They are given by

$$\begin{aligned} \tilde{\mathcal{A}}_{ab} &= \tilde{c}_{ab}\alpha_b^* - i\beta_b \int_{-\infty}^{\infty} dt'\, f_{bk}^*(t') \left[V_{bc}(t') \tilde{c}_{ac} f_{ck}^*(t') + W_{bc}^*(t') \tilde{c}_{ac}^* f_{ck}(t') \right], \\ \tilde{\mathcal{B}}_{ab} &= \tilde{c}_{ab}\beta_b^* - i\alpha_b \int_{-\infty}^{\infty} dt'\, f_{bk}^*(t') \left[V_{bc}(t') \tilde{c}_{ac} f_{ck}^*(t') + W_{bc}^*(t') \tilde{c}_{ac}^* f_{ck}(t') \right], \end{aligned} \tag{2.45}$$

where $\tilde{c}_{ab}$ must satisfy

$$\tilde{c}_{ab} - i\tilde{c}_{ac} \int_{-\infty}^{\infty} dt'\, V_{bc}(t') f_{bk}(t') f_{ck}^*(t') - i\tilde{c}_{ac}^* \int_{-\infty}^{\infty} dt'\, W_{bc}^*(t') f_{bk}(t') f_{ck}(t') = \delta_{ab}. \tag{2.46}$$

In the case of $W_{ab}(t) = 0$, for which the global $U(1)$ symmetry exists corresponding to the rotation of all $\chi_a(x)$ with a common phase, the scattering matrices are simply given by

$$
\begin{aligned}
\mathcal{A} &= N^{-1}\left(\alpha + K\beta^{\dagger}\right), & \mathcal{B} &= N^{-1}\left(\beta + K\alpha^{\dagger}\right), \\
\tilde{\mathcal{A}} &= \tilde{N}^{-1}\left(\alpha^{\dagger} + \tilde{K}\beta\right), & \tilde{\mathcal{B}} &= \tilde{N}^{-1}\left(\beta^{\dagger} + \tilde{K}\alpha\right),
\end{aligned}
\tag{2.47}
$$

where α and β are diagonal matrices defined by

$$
\alpha = \operatorname{diag}\left(\alpha_1, \alpha_2, \cdots, \alpha_n\right), \qquad \beta = \operatorname{diag}\left(\beta_1, \beta_2, \cdots, \beta_n\right),
\tag{2.48}
$$

and the matrices N, $\tilde{N}$, K and $\tilde{K}$ are given by

$$
\begin{aligned}
N_{ab} &= \delta_{ab} + i \int_{-\infty}^{\infty} dt\, V_{ba}(t) f_{ak}(t) f_{bk}^{*}(t), \\
\tilde{N}_{ab} &= \delta_{ab} - i \int_{-\infty}^{\infty} dt\, V_{ba}(t) f_{ak}^{*}(t) f_{bk}(t), \\
K_{ab} &= i \int_{-\infty}^{\infty} dt\, V_{ba}(t) f_{ak}(t) f_{bk}(t), \\
\tilde{K}_{ab} &= -i \int_{-\infty}^{\infty} dt\, V_{ba}(t) f_{ak}^{*}(t) f_{bk}^{*}(t).
\end{aligned}
\tag{2.49}
$$

$$
\tag{2.50}
$$

Because of the hermiticity of the matrix $V_{ab}(t)$, $\tilde{K}^{\dagger} = K$ and $T_{ab} \equiv \int_{-\infty}^{\infty} dt\, V_{ba}(t) f_{ak}(t) f_{bk}^{*}(t) = T_{ba}^{*}$, the matrix N can be regarded as the unitary matrix $\exp(iT)$.

3   Example

Once the model of the inflaton interacting with at least two complex scalars is specified, one can evaluate the 'effective potential' $V_{ab}(t)$ and $W_{ab}(t)$, which may be induced directly by coupling with the inflaton to the complex scalars, or through some loop effects including the scalars and any other fields. Here, without proposing any specific model, we illustrate calculation of the generated charges and particle numbers, assuming some simple form of the CP-violating potentials. For simplicity, we concentrate on the case of $W_{ab}(t) = 0$ and $n = 2$ and assume the degeneracy of the scalar masses $(m_1 = m_2)$ and the property of the potential $V_{11} = V_{22}$, which may result from some discrete symmetry. Under these assumptions,

$$
f_{1k}(t) = f_{2k}(t) = f_k(t), \qquad \alpha_1 = \alpha_2 = \alpha, \quad \beta_1 = \beta_2 = \beta.
\tag{3.1}
$$

Then the matrices in (2.49) and (2.50) are reduced to

$$
\begin{aligned}
K &= \begin{pmatrix} iK_1 & iK_2(1 - i\epsilon_K) \\ iK_2(1 + i\epsilon_K) & iK_1 \end{pmatrix}, & \tilde{K} &= K^{\dagger}, \\
N &= \begin{pmatrix} 1 + iN_1 & iN_2(1 - i\epsilon_N) \\ iN_2(1 + i\epsilon_N) & 1 + iN_1 \end{pmatrix}, & \tilde{N} &= N^{\dagger},
\end{aligned}
\tag{3.2}
$$

where

$$N_1 = \int_{-\infty}^{\infty} dt\, V_{11}(t)\, |f_k(t)|^2\,, \qquad N_2 = \int_{-\infty}^{\infty} dt\, \mathrm{Re} V_{12}(t)\, |f_k(t)|^2\,,$$

$$\epsilon_N = \frac{1}{N_2} \int_{-\infty}^{\infty} dt\, \mathrm{Im} V_{12}(t)\, |f_k(t)|^2\,, \tag{3.3}$$

$$K_1 = \int_{-\infty}^{\infty} dt\, V_{11}(t) f_k^2(t), \qquad K_2 = \int_{-\infty}^{\infty} dt\, \mathrm{Re} V_{12}(t)\, f_k^2(t),$$

$$\epsilon_K = \frac{1}{K_2} \int_{-\infty}^{\infty} dt\, \mathrm{Im} V_{12}(t)\, f_k^2(t). \tag{3.4}$$

Once the explicit forms of $f_k(t)$, $V_{11}(t)$ and $V_{12}(t)$ are provided, we can calculate all the quantities needed to follow the descent equations (2.27), so that the time evolution of the densities of particle number and charges.

First of all, we prepare the unperturbed wave which is appropriate for the perturbative calculation of the scattering matrices. For this purpose, we substitute the oscillating potential near its zeros with the asymptotically flat potential as

$$\Phi^2 \sin^2 mt \simeq 2\Phi^2 \tanh^2\left(\frac{m(t-t_j)}{\sqrt{2}}\right). \tag{3.5}$$

This is better than the simple quadratic potential adopted in [3], in that the potential of (3.5) matches to the original oscillating potential up to the forth order of $m(t - t_j)$. Introducing the dimensionless parameters

$$\tau = k_*(t - t_j), \quad \kappa = \frac{k}{k_*}, \quad \xi = \sqrt{\frac{m}{2g\Phi}} = \frac{1}{2q^{1/4}}, \tag{3.6}$$

where $k_* = \sqrt{g\Phi m} = \sqrt{2}mq^{1/4}$ and $q = \frac{g^2\Phi^2}{4m^2}$, the wave equation is expressed as

$$\frac{d^2 f_k(\tau)}{d\tau^2} + \left[\kappa^2 + \frac{1}{\xi^2}\tanh^2(\xi\tau)\right] f_k(\tau) = 0. \tag{3.7}$$

We find that the solution which asymptotically approaches the positive-frequency wave in the remote past is given, in terms of $x \equiv \xi\tau$, by

$$\begin{aligned} f_k(t) &= \frac{1}{\sqrt{2\omega_0}} \left(e^x + e^{-x}\right)^{-i\epsilon} F\left(a, b, c; \frac{1}{1+e^{-2x}}\right) \\ &\xrightarrow{\tau \to -\infty} \frac{1}{\sqrt{2\omega_0}} e^{-i\omega_0(t-t_j)}, \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} \epsilon &= \frac{g\Phi}{m}\sqrt{1 + \frac{k^2}{2g^2\Phi^2}} = 2q^{1/2}\sqrt{1 + \frac{\kappa^2}{4q^{1/2}}}, \\ \omega_0 &= 2\epsilon\xi k_* = \sqrt{2}\, m\epsilon, \end{aligned} \tag{3.9}$$

12

$$a = -4iq^{1/2}\left(\sqrt{1+\frac{\kappa^2}{4q^{1/2}}}-\sqrt{1-\frac{1}{64q}}\right)+\frac{1}{2},$$

$$b = -4iq^{1/2}\left(\sqrt{1+\frac{\kappa^2}{4q^{1/2}}}+\sqrt{1-\frac{1}{64q}}\right)+\frac{1}{2},$$

$$c = -4iq^{1/2}\sqrt{1+\frac{\kappa^2}{4q^{1/2}}}+1, \tag{3.10}$$

and $F(a,b,c;z)$ is the hypergeometric function. By use of its property, we find, in the limit of $t-t_j \to \infty$,

$$f_k(t) \longrightarrow \frac{\alpha_k}{\sqrt{2\omega_0}}e^{-i\omega_0(t-t_j)} + \frac{\beta_k}{\sqrt{2\omega_0}}e^{i\omega_0(t-t_j)}, \tag{3.11}$$

where

$$\alpha_k = \frac{\Gamma(-2i\epsilon+1)\Gamma(-2i\epsilon)}{\Gamma\left(\frac{1}{2}-2i\epsilon+4iq^{1/2}\sqrt{1-\frac{1}{64q}}\right)\Gamma\left(\frac{1}{2}-2i\epsilon-4iq^{1/2}\sqrt{1-\frac{1}{64q}}\right)},$$

$$\beta_k = \frac{\Gamma(-2i\epsilon+1)\Gamma(2i\epsilon)}{\Gamma\left(\frac{1}{2}+4iq^{1/2}\sqrt{1-\frac{1}{64q}}\right)\Gamma\left(\frac{1}{2}-4iq^{1/2}\sqrt{1-\frac{1}{64q}}\right)} = -i\frac{\cosh\left(4\pi q^{1/2}\sqrt{1-\frac{1}{64q}}\right)}{\sinh(2\pi\epsilon)}. \tag{3.12}$$

Among the phases defined in (2.18), θ_k^j is given by

$$\theta_k^j = \int_0^{t_j} dt\,\omega(t) = j\,\theta_k, \tag{3.13}$$

with

$$\theta_k = 2\sqrt{2}\,q^{1/4}\int_0^{\pi/2} d\varphi\sqrt{\kappa^2+2q^{1/2}\sin^2\varphi}, \tag{3.14}$$

while ϑ_k is evaluated by use of the potential (3.5) as

$$\vartheta_k = 2\epsilon\,\log\frac{\epsilon}{q^{1/4}\kappa} - 4q^{1/2}\log\frac{q^{-1/4}(\epsilon+2q^{1/2})}{\kappa}. \tag{3.15}$$

With these quantities, one can evaluate the density of the particle number in the absence of the perturbation. The result reproduces the same as that in [3], for the case of $q^{1/2} = 1/(4\xi^2) \gg 1$.

As for the effective potential, we adopt

$$V_{11}(t) = V_{22}(t) = 2\lambda_1\Phi^2(\tanh^2\xi\tau - 1),$$

$$V_{12}(t) = 2\lambda_2\Phi^2\,e^{i\theta(\tau)}(\tanh^2\xi\tau - 1), \tag{3.16}$$

which have the same t-dependence as the potential in (3.7) and vanish for large $|\tau|$. We will specify the CP-violating phase $\theta(\tau)$ later when we execute numerical calculation. If we denote

$$l_1 = \frac{\lambda_1}{g^2}, \qquad l_2 = \frac{\lambda_2}{g^2}, \tag{3.17}$$

13

the quantities composing the elements of the matrices (3.2) are given by

$$
\begin{aligned}
N_1 &= -\frac{2l_1 q^{1/2}}{\sqrt{1+\kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \, \frac{1}{\cosh^2 x} \left| F\left(a,b,c;\frac{1}{1+e^{-2x}}\right) \right|^2, \\
N_2 &= -\frac{2l_2 q^{1/2}}{\sqrt{1+\kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \, \frac{\cos\theta(x)}{\cosh^2 x} \left| F\left(a,b,c;\frac{1}{1+e^{-2x}}\right) \right|^2, \\
\epsilon_N &= -\frac{1}{N_2} \frac{2l_2 q^{1/2}}{\sqrt{1+\kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \, \frac{\sin\theta(x)}{\cosh^2 x} \left| F\left(a,b,c;\frac{1}{1+e^{-2x}}\right) \right|^2, \\
K_1 &= -\frac{2l_1 q^{1/2}}{\sqrt{1+\kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \, \frac{1}{\cosh^2 x} \left(e^x + e^{-x} \right)^{4i\epsilon} F\left(a,b,c;\frac{1}{1+e^{-2x}}\right)^2, \\
K_2 &= -\frac{2l_2 q^{1/2}}{\sqrt{1+\kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \, \frac{\cos\theta(x)}{\cosh^2 x} \left(e^x + e^{-x} \right)^{4i\epsilon} F\left(a,b,c;\frac{1}{1+e^{-2x}}\right)^2, \\
\epsilon_K &= -\frac{1}{K_2} \frac{2l_2 q^{1/2}}{\sqrt{1+\kappa^2/(4q^{1/2})}} \int_{-\infty}^{\infty} dx \, \frac{\sin\theta(x)}{\cosh^2 x} \left(e^x + e^{-x} \right)^{4i\epsilon} F\left(a,b,c;\frac{1}{1+e^{-2x}}\right)^2.
\end{aligned}
\tag{3.18}
$$

Now we are ready to evaluate the densities of the particle number and charges. The remaining quantities we must specify are the values of q, l_1 and l_2, and the functional form of $\theta(\tau)$. All the quantities with mass dimensions are expressed with appropriate powers of the inflaton mass m. For definiteness, we adopt

$$
\theta(x) = \frac{\theta_0}{1+e^{2x}},
\tag{3.19}
$$

which changes from θ_0 to 0 as x goes from $-\infty$ to ∞. We take $q = 200$, for which the parametric resonance predicted by the Mathieu function occurs for a wide range of the momentum κ, that is, in the broad resonance. We numerically evaluate the scattering matrices (2.47) for a fixed κ, and study the time evolution of

$$
\begin{aligned}
n_k^j &= \sum_{a,b=1}^{2} \left(\left| \beta_{ab}^j \right|^2 + \left| \tilde{\beta}_{ab}^j \right|^2 \right), \\
j_{1k}^j &= \left| \tilde{\beta}_{11}^j \right|^2 + \left| \tilde{\beta}_{21}^j \right|^2 - \left| \beta_{11}^j \right|^2 - \left| \beta_{21}^j \right|^2, \\
j_{2k}^j &= \left| \tilde{\beta}_{12}^j \right|^2 + \left| \tilde{\beta}_{22}^j \right|^2 - \left| \beta_{12}^j \right|^2 - \left| \beta_{22}^j \right|^2,
\end{aligned}
$$

$$
\tag{3.20}
$$
$$
\tag{3.21}
$$

where (3.20) and (3.21) are the charge densities defined with $Q_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $Q_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ in (2.15), respectively, by numerically solving the descent equations (2.27) with the initial condition $\alpha^0 = \tilde{\alpha}^0 = 1$ and $\beta^0 = \tilde{\beta}^0 = 0$. Because of the $U(1)$ symmetry

14

mentioned above (2.47), $j_{1k}^j + j_{2k}^j = 0$ in any j-th interval. Although the particle number grows exponentially for κ in one of the resonance band, it ceases to grow due to the back reaction after a finite number of inflaton oscillations, as studied in [3]. How many times the inflaton oscillates before the particle creation ends depends on q and the coupling constant g. For simplicity, we use the densities in the tenth interval when we evaluate the total densities of the generated particles and charges, since 10 is a typical number of the oscillations before the preheating ends.

Now let us show the numerical results. At first, neglecting the back reaction, we studied the time evolution of the densities for $\theta_0 = 10^{-3}$, $l_1 = 0.01$ and $l_2 = 0.02$. Since β_k in (3.12) behaves as $|\beta_k| \simeq e^{-\pi\kappa^2}$, the modes with κ satisfying $\kappa \lesssim 1/\sqrt{\pi} \simeq 0.56$ are expected to belong to the first resonance band. The time evolution of n_k and j_{1k} for several κ's are shown in Figs. 1 and 2. As expected, the densities for $\kappa \lesssim 0.5$ grow
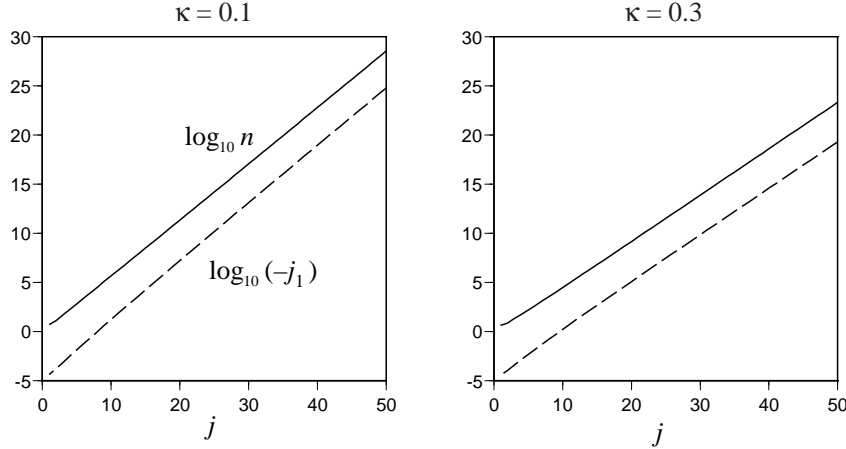

Figure 1: Time evolutions of the densities n_k (solid curve) and j_{1k} (dashed curve) for $\kappa = 0.1$ and 0.3.

exponentially, and $\left| j_{1k}^j \right| \sim 10^{-4} n_k^j$ for this choice of the parameters.

We estimate the total amount of densities of the generated particles and charges by integrating n_k^j and j_{1k}^j at $j = 10$ over k:

$$
\begin{aligned}
n &= \int d^3\boldsymbol{k}\, n_k = 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, n_k, \\
j_1 &= 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, j_{1k}, \\
j_2 &= 8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, j_{2k}.
\end{aligned}
\tag{3.22}
$$

We show the behavior of the integrands at $j = 10$ as functions of κ for several values of θ_0 in Fig. 3. Except for the case of $\theta_0 = \pi$, the generated particle numbers are not affected by the CP violation, while the generated charge is almost proportional to θ_0. As seen
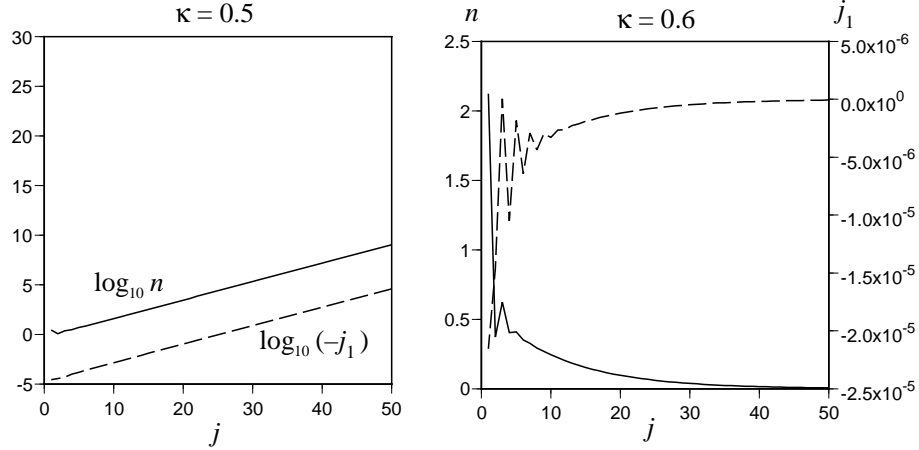
Figure 2: Time evolutions of the densities n_k (solid curve) and j_{1k} (dashed curve) for $\kappa = 0.5$ and 0.6.

θ_0	$\int d\kappa\, \kappa^2 n_k$	$\int d\kappa\, \kappa^2 j_{1k}$
10^{-3}	130.5096	-1.609334×10^{-2}
10^{-2}	130.5156	-1.544579×10^{-1}
10^{-1}	131.1163	-1.537716
π	990.7411	-50.84228

Table 1: The integrals appearing in the total densities of particle number and charges (3.22) for several values of θ_0.

from the κ-dependence of the integrands of (3.22) in Fig. 3, the second resonance band exists near $\kappa = 0.8$ but its contribution is about five orders smaller than that from the first resonance band. We performed the calculation up to $\kappa = 10$ and found several higher resonance bands, whose contributions are negligible. The values of the integrals in (3.22) are listed in Table 1.

We also calculated the total densities for various values of the coupling constants l_1 and l_2 for $\theta_0 = \pi$. As shown in Table 2, there is no regularities in how the total amount of densities depend on the coupling constants.

l_1	$\int d\kappa\, \kappa^2 n_k$	$\int d\kappa\, \kappa^2 j_{1k}$
10^{-4}	1.071055×10^5	2.456463×10^2
10^{-3}	6.773442×10^4	-2.316711×10^3
10^{-2}	9.907411×10^2	-5.084228×10^1
10^{-1}	1.656527×10^3	-2.069592×10^2

Table 2: The integrals appearing in the total densities of particle number and charges (3.22) for several values of l_1 and $l_2 = 2l_1$ with $\theta_0 = \pi$.
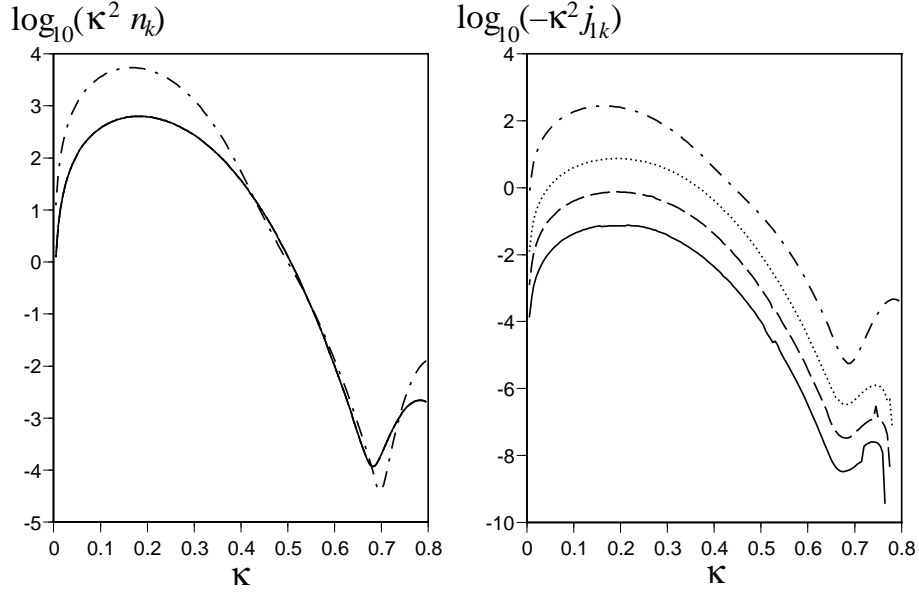
16

Figure 3: The densities of the particle number $\kappa^2 n_k$ and charge $-\kappa^2 j_{1k}$ at $j = 10$ for $\theta_0 = 10^{-3}$ (solid curve), 10^{-2} (dashed curve), 10^{-1} (dotted curve) and π (dashed-dotted curve).

4   Discussions

Since all the particles at present are thought to be created after the reheating process in the inflationary scenario and the process is far from equilibrium, it is natural to consider that the asymmetry between matter and antimatter was also generated in that era. We formulated charge generation during the preheating, which is viewed as a particle creation in the oscillating background.

If the scale factor of the FRW universe changes from $R_{\text{preheating}}$ to R_{th} just after thermalization, the charge-to-entropy ratio will be given by

$$\frac{j_1}{s} = \frac{8\sqrt{2}\pi\, m^3\, q^{3/4} \int_0^\infty d\kappa\, \kappa^2\, j_{1k}}{\frac{2\pi^2}{45} g_{*S} T_{\text{th}}^3} \left(\frac{R_{\text{preheating}}}{R_{\text{th}}}\right)^3, \tag{4.1}$$

where g_{*S} counts the degrees of freedom contributing to the entropy and T_{th} is the temperature just after thermalization. So far, we have not specified what the charge is, but we expect it to be conserved after the preheating process. One may think it as the baryon or lepton number, regarding the χ-field as the scalar partner of the quarks or leptons. If the preheating occurs before the electroweak phase transition, the generated lepton number is converted to the baryon number through the sphaleron process. When the charge is the weak hypercharge, the equilibrium state after thermalization with the hypercharge excess will become baryon-rich state, just as predicted in the theory of electroweak baryogenesis.

Anyway, our next subject will be to construct a realistic model, including at least two complex scalars, which admit the charge generation based on the mechanism studied here and to evaluate the baryon asymmetry produced by that model.

Although we have considered the coherently oscillating field as the inflaton, the formalism may be applicable to the Affleck-Dine scalar, which is coherently rotating in the internal space with a constant frequency if we neglect the damping due to the expansion of the universe. The AD scalar, which is the classical squark or a slepton field, is thought to decay perturbatively to quarks and leptons to leave B or L. This situation is similar to that of the conventional reheating process and one may expect that there is also a process similar to the preheating in the AD mechanism.

Acknowledgments

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